Forward to “Practical Isochronous Pendula(?)”

My interest in clockwork has been subliminal for most of my life. My first experience was when I was seven years old. My grandfather had a wind-up train with a circle of track on a green painted piece of plywood. I coveted it so much I was having dreams about it and its little puffs of smoke magically coming out of the stack. All trips to play outside went by way of the garage where it sat on sawhorses, begging me to play with it. Alas, I was instructed never to touch it.

I pressed and pressed, and finally my grandfather went to his bedroom, collected the alarm clock beside his bed (a large golden brass one with two large bells on the top), and sat me down with a couple of screwdrivers and said: “I’m tired of hearing about the train. When you can take this clock completely apart and put it back together so that everything works, you can have the train.” No help was forthcoming, and he continued on with his day.

I was forced to take meal breaks that day, and was the proud owner of the train by the next morning.

Another brush with clockwork was as a young aerospace designer. I had built an escapement that oscillated a dumbbell to rate limit the deployment of an extendable mast for a spacecraft, which I now know to be a Foliot, or as I thought at the time, just a great “energy eater”.

It was not until being recommended to *The Longitude*[1] in my early 40’s that I discovered a deeply neglected interest in precision clockwork. I have spent my adult life creating devices for precision motions, various mechanisms for spacecraft and large scale dynamic spacecraft models that only slightly pre-dated our complete confidence in finite element modeling of large structures in a weightless environment.

I definitely could relate to Harrison’s mind-set. I’ve since gone to Greenwich and watched his fantastic clocks running until the docent shooed me out. I am still in awe of the accomplishment and the tools to which Harrison was limited. Sadly, at the time I had no idea of the significance of the Shortt clock and others I may have glanced at on my way to see Harrison’s. Shortly thereafter, I got Philip Woodward’s *MORT*[2] and after devouring it in quick bites, was instantly, insatiably caught up in the state of the art, and his W5, and had the great pleasure of watching his amazing movement as interpreted by David Walters in his (D)W5 in the same month.

I’ve had an extremely pleasurable time with the Horological Science Newsletter back-issues, and realize that I’ve only scratched the diminishingly thin patina on the surface of research available to me, and in fact, as of this writing I am only up to 1999, but have read 2009. I must say that the HSN’s contributors are a fine bunch of scientists, and that the state of the art is consummately presented, and the math so beautiful and rendered understandable (though not always entirely by me), for the most part supported by data of test run durations that stagger my mind.

I’ve read of several ingenious methods of removing, mitigating or compensating for circular error here in HSN; too many to cite, and wanted to propose something original if at all possible. As I embarked on creating something of interest to this tough room, I am fully expecting responses such as *"so and so beat that to death in 1680! Next!"* That's OK, if so.

In the face of far more accurate electrical means, I find the free pendulum clock to be the ultimate expression of mechanism, and its optimization a profound riddle in mechanics and physics.

I had the pleasure of presenting an outline of the following work over dinner at David Walter's house, [after experiencing, photographing and filming my third (D)W5], with John Kirk in attendance. It was a pleasurable night of tech talk, over a wonderful single malt and Bolognese; talk that would leave most regular folk yawning and snatching surreptitious glances at wristwatches far more accurate than any I will likely create.

Why attempt a precision clock in the first place? Dallas Cain put it perfectly when he wrote: "the exercise has therapeutic benefits"[3] I could not agree more, and it is somehow much more. My life as a mechanical designer has to date always had a capitalistic bent. This is for the pure joy of it. I hope you like my take on the problem.

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Practical Isochronous Pendula(?)

Creating a perfectly cycloidal path for a pendulum bob as most will attest is no small task, and from what I can gather so far, not done well yet, and therefore the challenge of friendly competition with centuries of very creative inventors is quite motivating. Some would argue that the following walk through various cycloid generating mechanisms, rife with Q killing rollers and wrap bands, with pinch points that would not suffer a single skin cell in the wrong place; but I "*press on regardless"*. My impetus here is that true isochrony would eliminate some difficulty elsewhere in my clock project. Of one thing I'm sure; a more complete analytical treatment would have to come from more capable mathematicians than I. I can only get so far with graphical solutions, and clearly I'm not shy about sharing my early blatherings in a field new to me!

I began with mapping out some four-bars and slider-cam arrangements, eccentric pivots, and tautochrone ramps with rolling bob; coming quickly to the conclusion that a cycloid could or I should say, *must* be created using cylindrical and flat surfaces that can be made with highly exact profile and finish. While a grinding rig could be made fairly easily to cut this tautochrone path directly as a track, this approach's inelegance and preparatory work was less than appealing. As another resource I wanted to bring to bear, one of my patents deals with wrap band flexures with large excursions (for a flexure)[4], which are similar but different in a critical way to a Rolemite, which I see has had some interest here in HSN as a pendulum pivot means[5]. Bill Ellison in HSN 1994-5 reported on some experiments and put out a call for what the opposing surfaces look like in order to use a Rolemite to make an isochronic pendulum. I'll not be using a Rolemite, but it certainly could be used, and the surfaces turn out to be very simply: a known cylinder and a flat plane, just like you'd expect. But, I'm getting ahead of myself.

First out of the gate was to of course begin at the beginning: understand how is a cycloid generated, and discover if it can move from schematic representation to something real in a practical manner, eschewing the pitfalls of Huygen's "cheeks" or "chops", tricks of evolutes or other complex curvatures difficult to generate in a manufacturing sense.

Thanks to Christian Huygens we know that the proper cycloid for a given simple pendulum length, L_{sp}, is generated with a point on a circle, which is rolling "under a shelf", and whose radius, R_c= L_{sp} /4, Figure 1, (re-illustrated owing to countless references, with my own nomenclature). Of course what follows is applicable to any pendulum period desired, but I will be focusing in a hardware sense on a seconds beating pendulum, so R_c=248.41mm.

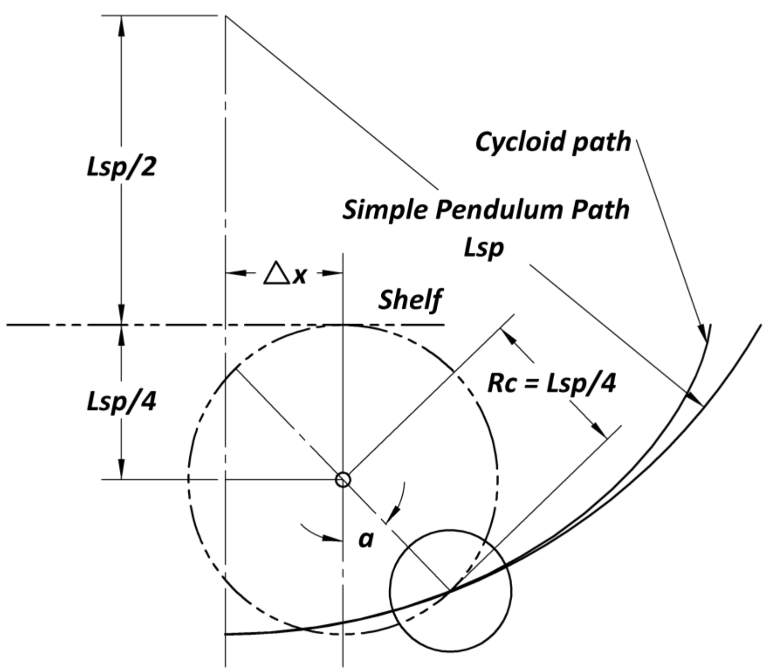

Figure 1, Cycloid generation basics

The basic element

What I consider the basic element is the simplest possible reduction of the schematic in Figure 1 to hardware, where a precision ground arc segment roll or shoe of radius R_c is lightly and magically held to, and rolls under an actual shelf, with rod extending down to a bob whose center is also on R_c. This makes a pendulum that is now half the length of the seconds beating simple pendulum it replaces, or 496.82mm, which sweeps a rather larger angle than it's predecessor for a given amplitude.

The method of holding this shoe to the underside of the shelf can be magnetic (Fig. 2a) or by way of a simple roller at the pivot point, Pc, running on top of another shelf (Fig. 2b). I originally spent a good deal of time imagining various configurations for the magnetic method, with my major concern being the elimination of extraneous forces in the X direction, which would directly turn into unwanted torque about Pc. I can imagine numerous effects degrading the consistency of the magnetic field, and think that maybe I could get mired with niggling issues with this approach. The mechanical roller has far fewer potential "gotchas", if one can accept sources of friction that directly degrade the pendulum Q. It is possible to use the bar magnet approach to off-load all but a few grams of the pendulum with the magnet above the support roller in Fig. 2b, if the roller were laminated steel disks, and there were no other ferromagnetic materials in the rest of the assembly. It's a general technique I'd like to employ somewhere here, to reduce contact stress and its attendant losses.

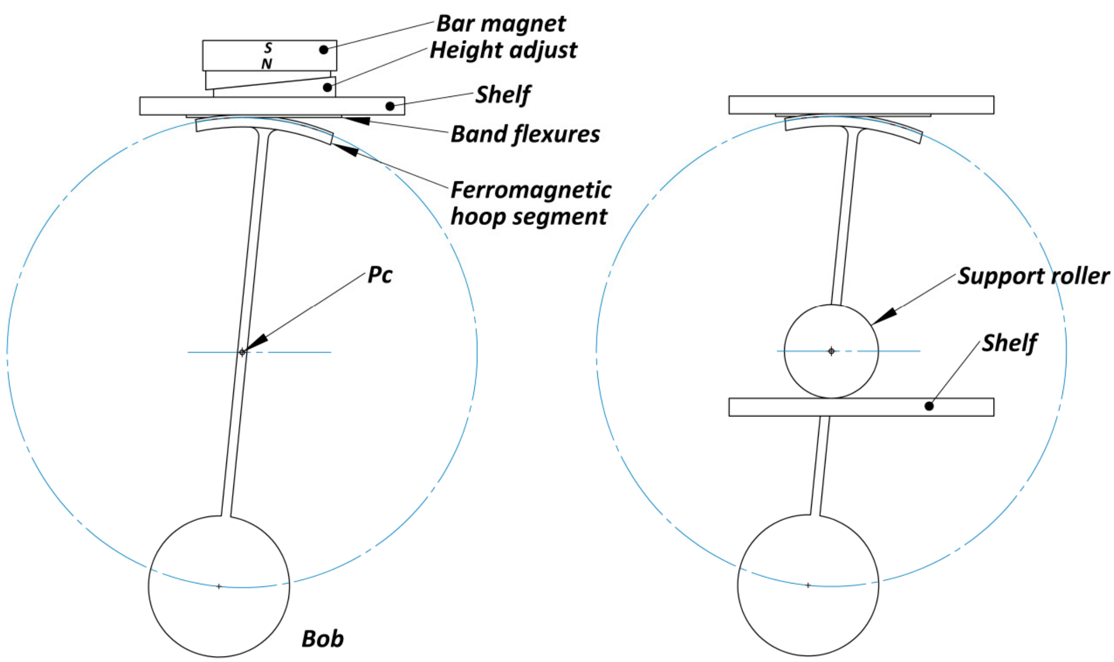

Figure 2a and 2b, The Basic Element with two support schemes

Both approaches above meet the main constraint, that the cycloid is generated with purely circular and planar surfaces which can be made with great exactitude. This would be something to see, I think, and no matter how much further you'll see me taking the reduction in pendulum length in what follows, 2a and 2b are the subject of hardware builds.

The subject of reducing the length of the pendulum while remaining a seconds-beating one is not a requirement, rather it is about making the cycloidal path perfectly with no mucking about. Length reduction to the point of conceptual *"Maximum Rediculosity"* is an interesting, perhaps not practical outcome of this exercise, yet it is the subject of the rest of this paper because it is faaaascinating to me (he said, stroking his goatee).

While any prototypes will display a reasonable effort toward thermal insensitivity, the primary thrust of the models will be show isochronism as an initial impulse decays, and the instrumentation used to prove it one way or another, and the fun of the exercise no matter how quickly they ultimately ring down.

Reducing the length by half again, or taking it to "Length Rediculosity Phase One"

The basic element said something to us about what needs to happen to the rod and bob, namely: as the rod segment immediately above the bob goes through angle α, its pivoting center (the center of the cycloid generating circle, P_c) must translate a distance $\Delta x = R_c \bullet \alpha$. Simple enough as identities go. A further critical observation, obvious as it may be, is that that center moves in a straight horizontal line. Understanding this, I then attempted folding the now half meter seconds pendulum rod at the virtual pivot point Pc, and considering how it must act if rolling now on *top* of a surface, coincident with the bob center instead of under a shelf, and how on earth to make the pendulum do what we want?

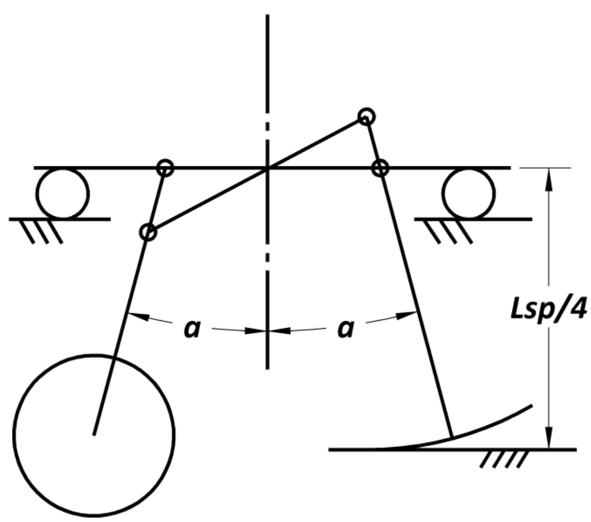

Figure 3, Folded Basic Element schematic

Our pendulum length now equals R_c (248.41mm). This again elicited attempts at straight line mechanisms to eliminate a linear slider for the pendulum pivot P_c, with the simple requirement that the pendulum rod must still rotate about P_c, but with an angle opposite in sign of the supporting rod and shoe, symmetrical about a vertical line. An "aha moment" came when I considered two support rods and shoes, spaced apart, connected by a link between their pivot centers, which would perform the straight line motion we need, and provide a pivot attach point for the pendulum, Figure 4. Looking ahead, the synchronizing links have been put in a configuration such that they can be pure tension members.

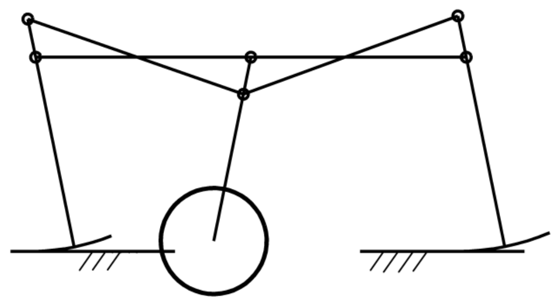

Figure 4, Dual Support Folded Basic Element

It should be noted that each supporting rod assembly must have a balance mass above its pivot point, Fig. 5, so there is no restoring torque toward vertical, save for that provided by the pendulum.

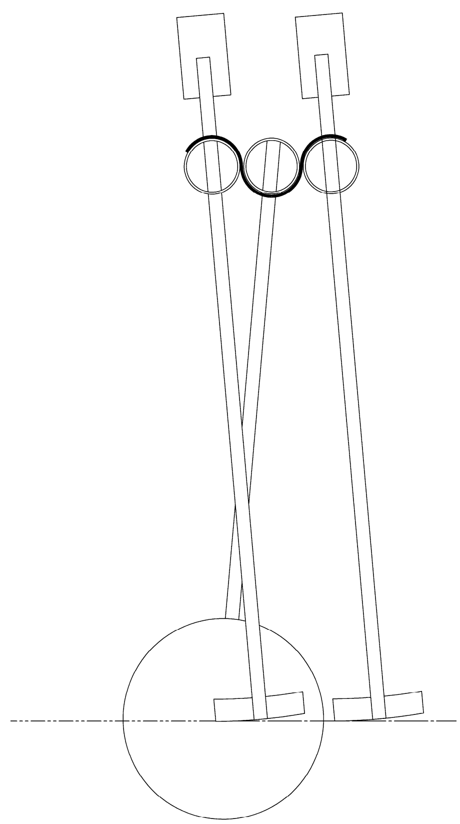

Figure 5, Dual support physical assembly

While the physical synchronizer links are accomplished with flexure bands, most likely of .001 to .0005" full-hard 301SS feeler stock, it is also possible to be straight links and jeweled pivots; the physical embodiment of the schematic links in Figure 4. I believe the bands are far easier to realize than tiny jeweled links, of which several are required, needing unreasonably equivalent lengths. This one would be fun to build and watch go; just don't push it too far! It may in fact be as far as I should take the progression, but let's continue on anyway.

Reducing the support legs to zero length

Why would we do such a thing? Because we can remove a length of material (the support legs) that must be thermally compensated, and simplify the assembly in general. Or just for the fun of it, as you prefer. As the flexure pivot is configured using flexure bands, we are presented with an opportunity to reduce the length of the support legs by adding a step up ratio between the pendulum pivot and the support leg pivots, adding no additional rolling or band flexure contact points. If this ratio is 1:2, then the support leg lengths can be reduced by half, and now sweep an angle that is 2α. The pivot system still now translates the required distance Δx, with the support plane now moved up half the distance to the pivot, Figure 6. A larger step up ratio allows us to delete the legs completely, moving the pivot support plane to the underside of the support rolls. Judicious material choice can make this a thermally stable section of the system.

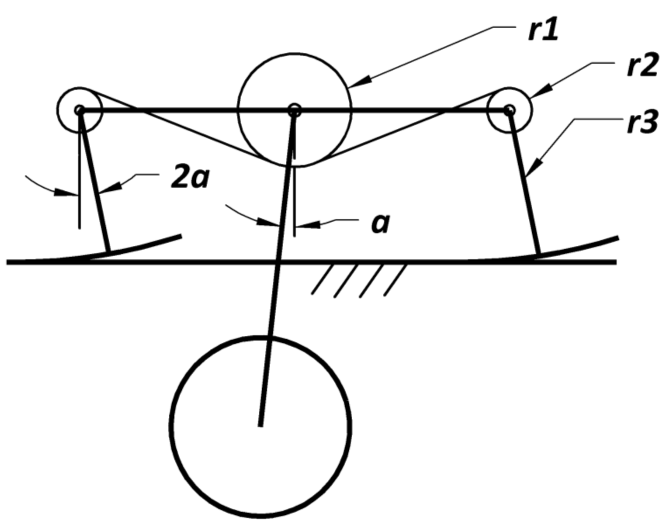

Figure 6, Applying a ratio to reduce support lengths

Pendulum pivot radius r_1 acts against support pivot radius r_2, creating ratio r_1/r_2 . The radius of the support roll that actually runs on the support plane is r_3 . The alphas in the following equation of course cancel.

$$\frac{r1}{r2} r3\, \alpha = \frac{Lsp}{4} \alpha$$

As an example, if we drive r_1 and r_3 to a radius of 50mm, the radius of r_2 is sought, so:

$$r2 = \frac{4\,r1\,r3}{Lsp} = \frac{10{,}000}{993.6} = 10.06mm$$

Really, we've not reduced the support legs to zero, but have reduced their effective length to r_3, in this case 50mm, removing the need for a physical rod. A further advantage is that the support rolls are now circular and inherently, or I should say easily balanced, eliminating the balancing weights above the support pivots, though of course these rolls must still be quite well balanced individually.

While this example gets us a reasonable flexure bend radius at r_2, the support rolls are rather large and ungainly (100mm diameter). If the pendulum pivot were pushed larger, r1 equal to 75mm for instance, becoming more of a "T" with arc segments at 9 and 3 o'clock, and r2 left at 10mm, then r3 can be of a more reasonable size:

$$r3 = \frac{r2\,Lsp}{4\,r1} = \frac{10 \cdot 993.6}{4 \cdot 75} = 33.12mm$$

Giving us a pendulum system that looks like this:

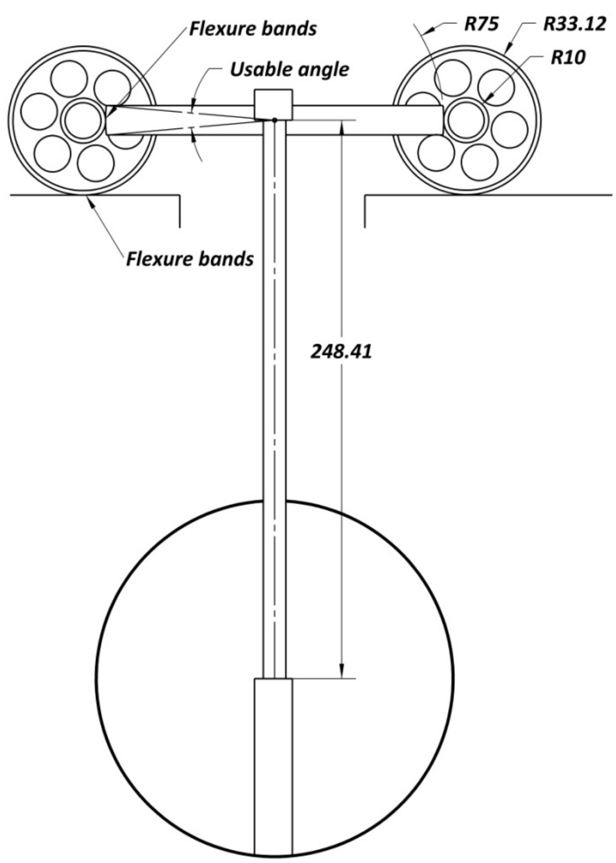

Figure 7, Isochronic seconds pendulum, closer to realistic

The physical embodiment of this is simplified over the design with support legs in Figure 5, though I expect it would have greater frictional losses.

Pendulum length reduction, taking it to "*Maximum Rediculosity*"

At least one sleepless night had me tossing about, certain that things could get simpler (if not more practical), and then it dawned on me to provide the bob mass with the critical radius Rc, tangent at its CG, and support it on idler rollers. This would eliminate all rods entirely, Fig. 7.

A property of idler rollers is that they can be of any diameter, creating no reduction or step-up between two surfaces, while reversing the surface translation of one to the other. So if the pendulum's CG were still on the radius Rc, any diameter rolls at any spacing would suffice between it and a flat surface, and the translation per angle α would be proper!

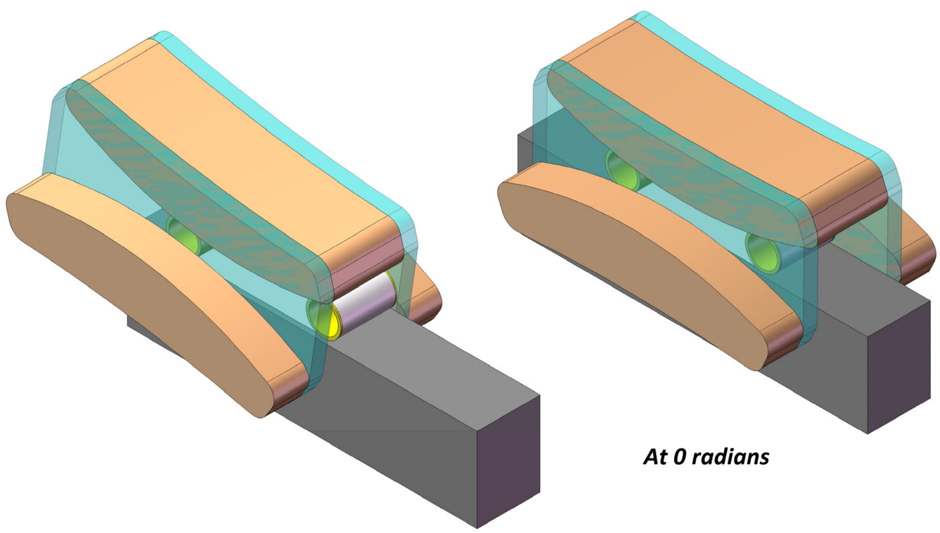

Figure 8, Rodless Assembly

While it's a simple matter to design the weights under the the CG to be proper, I've shown it made with weights that are basically the same part at half thickness on either side as a simple start on balancing. Again, while remotely possible to off-load the rollers magnetically, it will likely be fraught with problems.

A lower friction approach would be to provide a shoe between the support plane and the pendulum radius that has air-bearing surfaces, radial above, planar below, while keeping the flexure bands and rollers within to enforce the translation per rotation. While air-bearings are neat, viscous damping must be considered, and while I know my way around an air-bearing, it's not exactly the least expensive thing to try.

It should be noted that the flexure bands in all of these are purposefully wide in order to take a slight bit of shear (perpendicular to the plane of the pendulum), so that rate can be slowed by tilting the entire assembly out of the vertical plane; so the geometry must be made such that it is running a bit fast initially.

So, we've come full circle when I say, yes you can use a Rolemite in between these two easily generated surfaces.

Further work

It will be very fun to build one or two of these proposed pendula. I had originally wanted to generate data on one for inclusion in this paper, but obviously have opted to present the conceptual exercise as an initial salvo. Of course there is far more to add in order to have an actual timekeeper and if one of these pendula is in fact *all that*, then I hope I don't completely blow the escapement! Hopefully, some of the onus of escapement error at least can be removed with one of these methods.

As for my instrumentation approach, I'm planning for Microset, but for now will use what's at hand. One highly appropriate direct angular feedback method is putting 20μm optical encoder tape on a curved surface below the bob, and mounting a read head that will give me a non-contact arc-wise resolution from 1μm down to 5nm.

Most of what I've proposed has some unknowns that are frankly beyond my ability to analyze, like:

Are there moments in the short pendulum rod, Rc, which sweeps a larger angle than the simple pendulum rod it replaces, that detract from the desired goal?

Are my roller carriages and rods in fact exhibiting harmonic motion and how do or don't they affect the moment of inertia of the pendulum? Obviously, it's important to know if these ancillary bits must be extremely light, extremely low moment of inertia or not!

References:

[1] "The Illustrated Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time," Dava Sobel and William J. H. Andrewes, Jan. 2003

[2] "My Own Right Time: An Exploration of Clockwork Design," Philip Woodward. Sept. 1995

[3] "The Ultimate Pendulum: Higher Q or Better Drive? Part One," Dallas Cain. Horological Science Newsletter, NAWCC Chapter 161, 1995-1

[4] "Macro Motions with Macro Flexures with Emphasis on Vacuum Compatibility"
Everman, M. R. (Bell-Everman, Inc.) ASPE Spring topical meeting proceedings, May 2006
http://www.aspe.net/publications/Spring_2006/spr06abs/1875.pdf

[5] "SUSPENSION SYSTEM DESIGNED TO PROVIDE ISOCHRONOUS MOTION OF THE PENDULUM," Bill Ellison. Horological Science Newsletter, NAWCC Chapter 161, 1994-5

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Appendix regarding magnetic off-loading…. laminations applied to eliminate or reduce eddy current damping. There must also be a means of adjusting the magnetic attraction such that the shoe contact force is of only a few grams; just enough to hold it under the shelf, particularly when the bob reaches maximum velocity. This type of magnetic bearing unloading is currently of great interest to me in other than horological applications. It is necessary for this, though, as the pivots this competes with are of extremely low friction, and since all of these ideas involve some form of rolling Hertzian stresses, it must be minimized. In any case, I cannot expect a very high Q with any of these ideas, which is arguably (and I've read some of the arguments) not necessarily a bad thing.

Appendix regarding need for cleanliness…. In all cases, of course proper care must be taken to keep dust particles out of the contacting surfaces, and any of these pendula will have to be entirely covered after precision cleaning. The initial basic element approach has an advantage over all that follow in that it's running surface is upside-down and less likely to collect a dust particle.